Name:

**Calculus (Math 1545)**
**Practice Exam 1**

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This practice examination contains five problems which are worth 20 points each. It is essentially the first midterm exam from last year. The actual exam will look different than this one, as we have not covered the same topics.

No calculators or notes will be permitted.

| Prob 1 | Prob 2 | Prob 3 | Prob 4 | Prob 5 | Total Score |
|---|---|---|---|---|---|
| | | | | | |

**Problem 1. (Definitions)** Supply an appropriate short answer.

**(a)** Define $\log x$.

**(b)** Show that $\dfrac{d}{dx}\log x = \frac{1}{x}$.

**(c)** Show that $\log x$ is increasing, and so is invertible.

**(d)** Define $\exp x$.

**(e)** Show that $\dfrac{d}{dx}\exp x = \exp x$.

**(f)** Define $a^x$, where $a$ is a positive real number.

**Problem 2. (Differential Equations)** Solve the differential equation (best two of three). Remember to process the integration constants.

**(a)** $y' \exp(\tan y) = \cos^2 y$

**(b)** $y' + (\sec x)y = \cos x$

**(c)** $y''' = \dfrac{1}{x^2}$

**Problem 3. (Integration)** Find the antiderivative (best two of three).

**(a)** $\int \tan^3 x \, dx$

**(b)** $\int x^3 \log x \, dx$

**(c)** $\int \dfrac{dx}{x^4 - 16}$

**Problem 4. (Limits)** Evaluate the limit (best two of three).

**(a)** $\lim_{x\to 0}(e^x + x)^{1/x}$

**(b)** $\lim_{n\to\infty} \sqrt[n]{5^{2n+1}}$

**(c)** $\lim_{n\to\infty} \sum_{i=0}^{n} \dfrac{5^i - 3^i}{7^i}$

**Problem 5.** An *algebraic function* of $x$ is a function which is written using $x$, numbers, addition, subtraction, multiplication, division, and radicals.

**(a)** Express $\sin(\arctan(x))$ as an algebraic function of $x$.

**(b)** Evaluate $\int \sin(\arctan x)\, dx$.